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# Latency correction explains the classical geometrical illusions

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**Abstract.** There is a significant delay between the time when light hits the retina and the time of the consequent percept. It has been hypothesized that the visual system attempts to correct for this latency by generating a percept representative of the way the world probably is at the time the percept is elicited, rather than a percept of the recent past. Here we show that such a 'perceiving the present' hypothesis explains a number of classical geometrical illusions: the Hering, Orbison, Müller-Lyer, Double Judd, Poggendorff, Corner, and Upside-down-T illusions. Each stimulus is perceived as it would project in the next moment were the observer moving through the scene the stimulus probably represents. We also examine one general class of predictions made by the hypothesis, and report psychophysical experiments confirming the predictions.

#### 1 Introduction

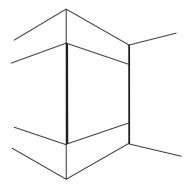
One might expect that it would be advantageous for humans to have visual percepts that accurately reflect reality. Such veridical perception is difficult to achieve, however, because visual percepts are elicited on the order of magnitude of 100 ms after the time light hits the retina (Lennie 1981; De Valois and De Valois 1991; Maunsell and Gibson 1992; Schmolesky et al 1998), and by then the world or the observer's position within it has often changed. It has been hypothesized that the visual system attempts to correct for this latency: rather than generating percepts of the way the world probably was roughly 100 ms before, the visual system generates percepts representative of the way the world probably is at the time the percept is actually generated (De Valois and De Valois 1991; Nijhawan 1994, 1997, 2001; Berry et al 1999; Khurana et al 2000; Schlag et al 2000; Sheth et al 2000; Changizi, in press; latency correction is under debate: Baldo and Klein 1995; Khurana and Nijhawan 1995; Lappe and Krekelberg 1998; Purushothaman et al 1998; Whitney and Murakami 1998; Krekelberg and Lappe 1999; Brenner and Smeets 2000; Eagleman and Sejnowski 2000; Khurana et al 2000; Whitney and Cavanagh 2000; Whitney et al 2000). Changizi (2001) used this 'perceiving the present' hypothesis to explain some simple cases of misperceptions of projected angle and projected size. Our primary goal in this paper is to demonstrate how the same hypothesis explains many of the classical geometrical illusions, including the Orbison, Hering, Poggendorff, Corner, Müller-Lyer, Double Judd, and Upside-down-T illusions (see figure 4). The paper is structured as follows. In section 2 we present a criticism of the most widely accepted hypothesis used to explain the classical illusions; we refer to this theory as the traditional inference approach. In section 3 we show how the latency-correction hypothesis explains the classical geometrical illusions as well as some predicted novel illusions. In section 4 we present a set of psychophysical predictions made by the theory, and report experimental results conforming well to the predictions.

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## 2 The traditional inference approach

One of the most venerable and well-entrenched functional theories of the geometrical illusions is what we will call the traditional inference approach (Helmholtz 1867/ 1962; Gregory 1963, 1997; Rock 1975, 1983, 1984; Gillam 1998; Nundy et al 2000), also sometimes referred to as constancy scaling (Gregory 1963, 1997). Before stating what the general form of this kind of theory is, we must distinguish between two kinds of perception. The first kind of perception concerns the properties of objects in the world independent of the observer's position, eg the perception of the angle between two tree branches, or the perception of the height of a tree. The second kind of perception concerns the manner in which objects in the world project toward the observer's eye, eg the perception of the angle that a pair of tree branches projects toward the eye, or the perception of the projected size of (or how much of the visual field is filled by) a tree. A number of terms have been used to mark this distinction, including 'objective' versus 'projective' (Gillam 1998), 'pictured three-dimensional scene' versus 'picture surface' (Sedgwick and Nicholis 1993), 'distal mode' versus 'proximal mode' (Palmer 1999), 'visual world' versus 'visual field' (Gibson 1950), 'constancy' versus 'proximal' (Mack 1978), and 'world' versus 'proximal' (Rock 1983). We favor the first of these, and will write 'objective angle' and 'objective size' to refer, respectively, to the real-world angles and sizes of objects; and we will write 'projected angle' and 'projected size' to refer, respectively, to the projected angles and projected sizes of objects.

By way of introducing the traditional inference approach to explaining the geometrical illusions, consider figure 1, where observers perceive the bold vertical line on the right to have greater projected size than the bold vertical line on the left; this is the illusion. Note the observers *also* perceive the objective size of the line on the right to be greater; that is, they perceive that it is a taller object in the depicted scene, when measured by a ruler in, say, meters. But this latter perception of objective size is not what is illusory about the figure: no one is surprised to learn that observers perceive that the line on the right has greater objective size in the depicted scene. What is illusory is that observers perceive the line on the right to have greater projected size—to fill more of the visual field—than the line on the left, despite their projected sizes being identical.



**Figure 1.** An illusion which is a variant of the Müller-Lyer illusion. The two bold vertical lines are the same projected size, but the right-hand one appears to have greater projected size.

The traditional inference explanation for this illusion states that the line on the right is perceived to be longer because the cues suggest that it probably *is* longer. Describers of the theory will usually also say that such a perception is useful for us in the real-world scene version of figure 1—ie when you are standing in front of a real hallway—but when the stimulus is from a piece of paper as it *actually* is in this figure, this perceptual strategy is said to become "inappropriate". There is, however, a deep conceptual problem with this explanation. To start, let us look again at the main statement, which is along the lines of:

The line on the right is perceived to be longer because the cues suggest that it probably is longer.

What does the statement mean by 'longer'? The first possibility is that it means 'greater objective size'. That is, the statement would be:

The line on the right is perceived to have greater objective size (eg in meters) because the cues suggest that it probably *is* greater in objective size.

The statement in this case would be fine, as far as it goes, since it is certainly useful to perceive the objective size to be what it probably is. For example, if the line on the right is probably 3 m high, then it is appropriate to perceive it to be 3 m high. However, this interpretation is no longer relevant to the illusion, since the illusion concerns the misperception of their projected sizes.

The second possible interpretation is that 'longer' means 'greater projected size', in which case the statement becomes:

The line on the right is perceived to have greater projected size (measured in degrees) because the cues suggest that it probably *is* greater in projected size.

This, however, is inadequate because the cues do *not* suggest that the line on the right has greater projected size. The lines have, in fact, identical projected size, and unambiguously project with identical projected sizes onto the retina.

So far, the traditional inference explanation statement is either irrelevant (the first interpretation) or false because the cues do *not* suggest that the line on the right has greater projected size (the second interpretation).

The third and final possible interpretation we will consider is that the first occurrence of 'longer' is interpreted as 'greater projected size' and the second occurrence of 'longer' is interpreted as 'greater objective size'. That is, in this possibility the statement is equivocating between two meanings of 'longer'. The statement is now:

The line on the right is perceived to have greater projected size (measured in degrees) because the cues suggest that it probably is greater in objective size (bigger in meters).

This appears to be the interpretation that people actually have, at least implicitly, when they state this view. It is sometimes even phrased as something along the lines of "the perception of the projective properties of the lines are biased toward the probable objective properties of the lines". The statement is not irrelevant as in the first interpretation; this is because the claim concerns the perception of projected size, which is what the illusion is about. The statement also does not err, as in the second interpretation, by virtue of claiming that the line on the right probably has greater projected size. One preliminary problem concerns what it could possibly mean to bias a projective property toward an objective property; how can something measured in degrees get pushed toward something that is measured in, say, meters? Another issue concerns how much the projected size should be increased in the probably-objectively-longer line; there is no theoretical apparatus providing an answer to this.

We will focus on another problem, which concerns the supposed *usefulness* of such a strategy for vision: of what possible use is it to perceive a greater projected size merely because the objective size is probably greater? The goal of the visual system according to these traditional inference approaches is to generate useful percepts, and, in particular, to generate percepts that closely represent reality (because this will tend to be useful). To represent accurately the projected sizes in figure 1 would be to perceive them as being identical in projected size. The visual system would *also* want to perceive them as having different objective sizes, but there is no reason—at least none that this traditional inference explanation gives—for the visual system to misperceive the projected sizes.

It is sometimes said that the illusion is only an illusion because figure 1 is just on a piece of paper. The inferential strategy of increasing the perceived projected size of the line on the right because it is probably objectively longer is inappropriate in this case because, it is said, the figure is just a figure on a page, where the lines in fact have the same objective size. If, the argument continues, the proximal stimulus were, instead, due to a real live scene, then the strategy would be appropriate. Unfortunately, the strategy would be inappropriate in this latter scenario too. To see this, let us imagine that the stimulus is not the one in figure 1, but, instead, you are actually standing in a hallway of the kind depicted, and your eye position is placed in just such a manner that the line on the right has the same projected size as the one on the left. Is there anything 'appropriate' about perceiving the line on the right to have greater projected size merely because its objective size is probably greater? It is not clear what would be useful about it, given that its projected size is the same as that of the line on the left, and perceiving their projected sizes to be equal does not preclude perceiving their objective sizes to differ. (For another example, hold your finger out until it fills just as much of your visual field as a tree off in the distance. You now perceive their projected sizes to be identical, but you also perceive the tree to be objectively larger.)

## 3 Explaining the geometrical illusions

In this section we explain how the latency-correction hypothesis explains the classical geometrical illusions. In section 3.1 we answer the question: what is the probable scene underlying each geometrical stimulus? This includes determining what the lines are and where they are with respect to the observer's direction of motion. In section 3.2 we look at the geometrical illusions that are misperceptions of projected angle, which include the corner, Poggendorff, Hering, and Orbison illusions. In section 3.3 we explain the illusions of projected size or projected distance, which include the Double Judd, Müller-Lyer, Hering, Orbison, and Upside-down-T illusions.

3.1 *The probable scene and observer direction of motion* Recall that the latency-correction hypothesis is as follows:

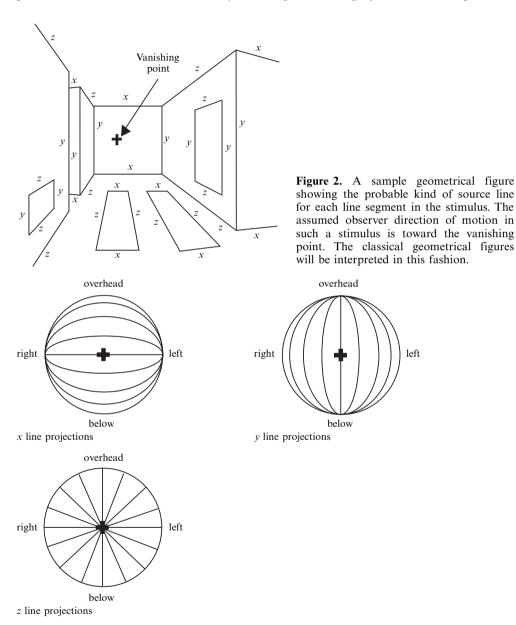
On the basis of the retinal information, the visual system generates a percept representative of the scene that will probably be present at the time of the percept.

[Note that we are not making any claim about how the visual system might implement latency correction. Also, note that there is no implication that observers should actually perceive motion from a static stimulus; observers should just perceive the scene that would probably be present by the time the percept occurs. If the stimulus is unchanging, then the elicited percept will always be the same.] Changizi (2001) put forth a 'carpentered world model' as a simplification which allowed us to make predictions from the latency-correction hypothesis. The central assumption is that there are predominantly the following three kinds of line in our experiences:

- x lines are the lines that lie parallel to the ground, and perpendicular to the observer's direction of motion.
- y lines are the lines that lie perpendicular to the ground, and are also perpendicular to the observer's direction of motion.
- z lines are the lines that lie parallel to the ground, and are also parallel to the observer's direction of motion.

Such lines are called *principal lines*, and are depicted in figure 2. [Note that we expect this model to apply only to people living in carpentered environments; those living elsewhere do not, in fact, appear to experience the classical geometrical illusions in the same manner (Segall et al 1966).]

The manner in which principal lines project towards an observer was discussed in Changizi (2001, pages 197-198), and may be understood by considering projection spheres on which x, y, and z lines have been projected (see figure 3). From this it is possible to determine several rules by which, given some projected line in a proximal



**Figure 3.** A projection sphere allows us to visualize the way things in the world project toward an observer. Projections are, by definition, devoid of depth information; they possess only information about the direction from which the stimulus was received. The set of all possible such directions from the outside world toward the observer's eye can be encapsulated as a sphere with the observer's eye at its center; each point on the sphere stands for a different projection direction from the outside world. This figure shows how the three kinds of line may project toward the observer within our simple model. Note that each of these figures depicts a sphere (even the z-line one), and the contours are on the near surface. The three projection spheres show, respectively, how x lines, y lines, and z lines project toward an observer. The focus of expansion is shown as the cross.

stimulus or figure, the probable source line can be determined (see Changizi 2001, page 198). Here we record these rules, but in more detail.

Rule 1: If there is a single set of oblique projected lines sharing a vanishing point, then their sources are probably z lines.

Rule 2: A horizontal projected line that does not lie on the horizontal meridian is probably due to an x line.

Rule 3: A horizontal projected line that does lie on the horizontal meridian may be due either to an x line or to a z line.

Rule 4: A vertical projected line that does not lie on the vertical meridian is probably due to a *y* line.

Rule 5: A vertical projected line that does lie on the vertical meridian may be due either to a y line or to a z line.

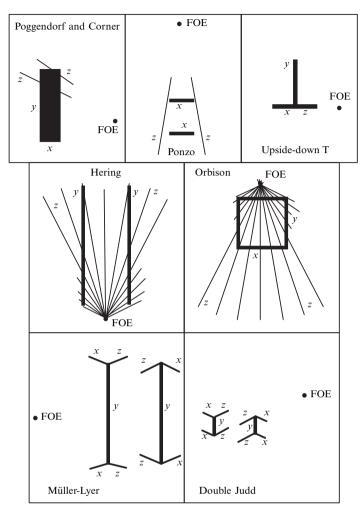
Rule 6: When there are two sets of projected lines with different vanishing points, the set with the more salient vanishing point probably consists of projections of z lines, and the other of either x or y lines, depending on where they point.

Rule 7: The probable location of the focus of expansion is the vanishing point of the projected z lines.

[One important aspect of the probable scenes that this simple model does not accommodate is distance from the observer. If all the probable sources were as in the model, but were probably a mile away, then we can expect no change in the nature of the projections in the next moment. It is reasonable to assume that latency correction will be primarily tuned to nearby objects, objects that we can actually reach, or that we might actually run into. Accordingly, it is plausible that the visual system interprets these geometrical stimuli as scenes having a distance that is on the order of magnitude of meters away, rather than millimeters of hundreds of meters (see also Cutting and Vishton 1995).]

These rules can now be applied to the illusions from figure 4, both in determining what are the probable sources of the stimuli, and in determining what is the probable direction of motion for the observer. Each projected line in figure 4 has been labeled with the probable kind of source line as determined by the rules. The explanations for the probable sources are as follows.

- No vertical line in any of the illusory figures has cues suggesting it lies along the vertical meridian, and thus each is probably due to a y line.
- Of all the horizontal lines, only the one in the Upside-down-T illusion possesses a cue that suggests it might lie along the horizontal meridian. The cue is that there is a T junction, and such junctions are typically due to three-dimensional corners (ie x-y-z corners). The horizontal segment of the T junction is probably, then, due to two distinct segments, one the projection of an x line, and one the projection of a z line. That is, it is probably a corner that is being viewed 'from the side'. We have arbitrarily chosen the left segment to be the projection of an x line, but the cues in the Upside-down-T illusion (which consists of just the upside-down T) do not distinguish which is which.
- All the remaining horizontal projected lines are parts of stimuli without any cues to suggest that they lie along the horizontal meridian, and so are thus due to x lines.



**Figure 4.** Eight classical geometrical illusions. Corner Poggendorff: the line through the corner of the rectangle appears to be bent. Poggendorff: the line through the rectangle appears to be two, parallel, noncollinear lines. Ponzo: the higher horizontal line appears to be longer than the same-length lower one. Upside-down T: the horizontal bar appears to be shorter than the same-length vertical bar resting on top of it. Hering (also a variant of the Zöllner stimulus): the two parallel lines appear to be farther apart as one looks lower. Orbison: the right angles near the top appear to be acute, and the right angles at the bottom appear to be obtuse. Müller-Lyer: the vertical shaft on the left appears longer than the same-length one on the right. Double Judd: the vertical shaft of the left figure appears higher than the same-height one on the right. [See Coren and Girgus (1978) for references; see Greene (1988) for the Corner Poggendorff.] Also, for each projected line the probable kind of source line—x, y, or z—is shown, as well as the approximate probable location of the focus of expansion (FOE).

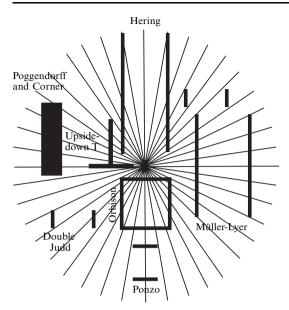
- All that is left are the obliques. In the Hering, Orbison, Ponzo, Corner, and Poggendorff illusions there exists just one set of converging obliques, and they are thus probably due to z lines.
- In each of the Müller-Lyer and the Double Judd illusions there are two sets of converging projected lines: one set consists of the four inner obliques (the ones in between the two vertical lines), and the other set consists of the four outer obliques (the ones not in between the two vertical lines). The four inner obliques are more salient and clustered, and appear to share a vanishing point more clearly

than do the outer ones. The inner obliques are therefore probably due to z lines. Since the outer obliques have a vanishing point horizontally displaced from the vanishing point for the inner obliques, the outer obliques must be due to x lines. [Although this serves as an adequate first approximation, greater analysis in fact reveals that the outer obliques probably do not share a vanishing point at all (and thus they cannot all be principal lines). Consider just the Müller-Lyer figure for specificity. Lines in the world project more obliquely as they near their vanishing point (see figure 3). The two outer obliques on the left are far in the visual field from the two outer obliques on the right; if they were projections of the same kind of line in the world, then they would not project parallel to one another, one pair being considerably closer to the vanishing point (for that kind of line) than the other. But the outer obliques on the left are parallel to the outer ones on the right, and thus they cannot be projections of the same kind of line, and they do not point to a single vanishing point. Only the four inner obliques are approximately consistent with a single vanishing point.]

Now that we know what the probable sources are for the eight illusory proximal stimuli, we can use the information about the projected z lines to determine the focus of expansion. That is, the z line vanishing point is the focus of expansion. Figure 4 also shows where each stimulus probably lies with respect to the focus of expansion.

- For the Hering, Ponzo, Orbison, and Müller-Lyer stimuli there is exactly one focus of expansion determined by the projections of the z lines, and figure 4 shows this. Also, figure 5 shows the key features of these figures embedded in a radial display at the appropriate location with respect to the probable focus of expansion (ie the vanishing point). Notice that for the Müller-Lyer stimulus the fins act as cues as to the location of the focus of expansion, and that in figure 5, where the radial display does the cueing work, the fins are no longer necessary for the illusion.
- The projected z lines for the double Judd stimulus are so similar in orientation that they may converge either up and to the right of the figure, or down and to the left of it; that is, the focus of expansion may be in one of these two spots. Figure 4 shows just one of these. The fin-less version of the double Judd illusion has been placed in figure 5 into these two positions with respect to the focus of expansion. Note that the illusions are qualitatively identical in each case to the earlier one (since the cues to the focus of expansion are provided by the radial display rather than by the fins).
- The Corner and Poggendorff illusions could have the focus of expansion placed anywhere so long as the projected z line is at the same angle; one spot has been chosen arbitrarily in figures 4 and 5. Any conclusions drawn later will not depend on this choice.
- The Upside-down-T illusion could be placed on either side of the vertical meridian (with respect to the focus of expansion), so long as the horizontal segments lie along the horizontal meridian. One spot has been arbitrarily chosen in figures 4 and 5. Any conclusions drawn later will not depend on this choice.

Recall that, under the latency-correction hypothesis, in addition to determining the probable scene causing the proximal stimulus—which is what we have done thus far—we must also figure out how that scene will probably change by the time the percept occurs. Since we know the probable scene, and we know which direction the observer is probably moving, all we have to do is to determine how the sources will project when the observer is moved forward a small amount.



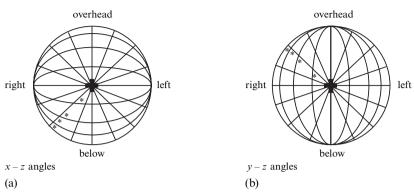
**Figure 5.** The misperceived segments from the classical illusions in figure 4 have been placed in a radial display in such a way that the center of the radial display is at the position of the probable focus of expansion for each figure. Because the radial display is now serving to cue the probable location of the focus of expansion, we expect, and indeed find, the same illusions as in figure 4. This suggests that it is cues to the location of the focus of expansion that is of primary importance in the illusions. In the case of the Double Judd and Müller-Lyer figures, for example, the radial display does the work that the fins did in figure 4. Note that because the Double Judd stimulus is also consistent with being in the upper right quadrant, it has been placed there as well as in the bottom left quadrant. The Corner and Poggendorff stimuli could be placed anywhere in the radial display so long as radial lines traverse them in the appropriate fashion.

#### 3.2 Projected-angle misperception

The Corner, Poggendorff, Hering, and Orbison illusions can be treated as misperceptions of projected angle. In the Corner and the Poggendorff illusions the angles appear to be nearer to  $90^{\circ}$  than they actually are. The same is true for the angle between the vertical line and the oblique lines in the Hering illusion. In the Orbison illusion, the right angles appear to be bent *away* from  $90^{\circ}$ . How do we make sense of these projected-angle illusions? And why are some misperceived towards  $90^{\circ}$  and some away from it?

First, following Changizi (2001), let us distinguish between two kinds of projected angle. Since there are just three kinds of line in the model, the only kinds of angle are those that result from all the possible ways there are to intersect these kinds of line. They are the x-y, x-z, and y-z angles; these are the *principal angles*. That is, x-y angles are any angles built from an x line and a y line, and so on. The x-z and y-z angles are actually similar in that, because they have a z arm, the plane of these angles lies parallel to the observer's direction of motion. We call x-z and y-z angles 'xy-z angles.' The x-y angles, on the other hand, lie in a plane perpendicular to the observer's direction of motion, and must be treated differently.

3.2.1 Projected xy-z angles. Note that the Corner, Poggendorff, and Hering illusions have angle misperceptions where the angles are xy-z angles (see figure 4), and the misperception is that observers perceive the projected angles to be nearer to 90° than they actually are. Why is this? The latency-correction hypothesis says it is because in the next moment the angles will project nearer to 90°, and thus the misperception is typically a more veridical percept (but is inappropriate in the case of a static stimulus on the page). But do xy-z angles actually project closer to 90° in the next moment?

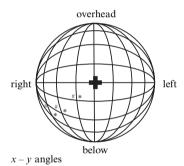


**Figure 6.** (a) A projection sphere upon which x and z lines are projected; their intersections are x-z angle projections. Notice how, along any radial line, the angles of intersection between x and z lines become more like  $90^\circ$  in the periphery (see the asterisks); that is how they change in the next moment, since the angles move radially toward the periphery as the observer moves forward. (b) A projection sphere upon which y and z lines are projected; their intersections are y-z angle projections. Notice how, along any radial line, the angles of intersection between y and z lines become more like  $90^\circ$  in the periphery (see the asterisks); this is how they change in the next moment. Thus, xy-z angles are predicted to perceptually 'regress' (Thouless 1931) toward  $90^\circ$ .

Yes, and this was demonstrated in Changizi (2001), and can also be comprehended by examining projection spheres upon which xy - z angles have been projected (see figure 6).

The Poggendorff stimulus has another salient illusory feature in addition to the projected angles being perceived nearer to 90° than they are: the two oblique lines are collinear, but do not appear to be. Each oblique line appears, intuitively, to undershoot the other. Latency correction explains this illusory feature as follows. Suppose that a single z line lies above you and to your left along the wall (perhaps the intersection between the wall and the ceiling). Now also suppose that there is a black rectangle on your upper left, but lying in your frontoparallel plane. That is, the rectangle is made of x and y lines. Suppose finally that the rectangle is lying in front of the z line. The projection of these objects will be roughly as shown by the Poggendorff illusion in figure 4. We say "roughly" because the projection will not, in fact, be as in this figure. Consider first the projected angle the z line will make with the right side of the rectangle. Suppose it is  $60^{\circ}$ ; that is, the (smaller) y-z angle on the right side of the rectangle is 60°. What will be the projected angle between the same z line and the other vertical side of the rectangle? The part of the z line on the other vertical side of the rectangle is farther away from the focus of expansion and more in your periphery. Thus, this more peripheral y-z angle will be nearer to 90°; let us say 63° for specificity. That is, when the same z line crosses through or behind a rectangle as constructed, the projected angles will not be the same on either side. Now, the two projected angles in the Poggendorff figure are the same on either side, and thus the projected lines on either side cannot be due to one and the same z line. Instead, the more peripheral y-zprojected angle, being farther from 90° than it would be were it to be the projected angle made with the z line from the other side, must actually be due to a line that is physically higher along the wall. The visual system therefore expects that, in the next moment (ie by the time the percept is generated), the oblique projected line on the left should appear a little higher in the visual field compared with the extension of the oblique line on the right (since differences in visual-field position are accentuated as an observer moves forward).

3.2.2 Projected x - y angles. The Orbison illusion primarily concerns the misperception of the four projected angles, each of which is 90°, but which observers perceive to be greater or lower than 90°. The squares in the Orbison illusion are composed of x and y lines (figure 4), and we must ask how the projections of x - y angles change as observers move toward the focus of expansion (which is the vanishing point of the projected z lines in the Orbison figure). Changizi (2001) demonstrated the manner in which x - y angle projections change when an observer moves forward: x - y angles project further away from 90° in the next moment; they are 'repulsed' away from 90° instead of regressed toward  $90^{\circ}$  as in xy-z projected angles. Furthermore, the direction in which a projected x - y angle will get pushed away from 90° depends on the orientation of the angle and its position relative to the focus of expansion. This may also be understood by examining a projection sphere on which x and y lines have been projected, as shown in figure 7. The latency-correction hypothesis therefore predicts that if cues suggest that a projected angle is due to an x-y angle, then observers will misperceive the angle to be whatever it will probably be in the next moment (by the time the percept is elicited). Figures 6 and 7 of Changizi (2001) show that observers misperceive projected x - y angles as predicted, and the Orbison illusion is just a special case.



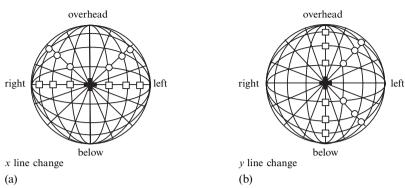
**Figure 7.** A projection sphere upon which x and y lines are projected; their intersections are x-y angle projections. Notice how, along any radial line, the angles of intersection between x and y lines become less like  $90^\circ$  in the periphery (see the asterisks and hash signs); that is how they change in the next moment since the angles move radially toward the periphery as the observer moves forward.

## 3.3 Projected size misperception

We have now seen that the illusions of projected angle—the Corner, Poggendorff, Hering, and Orbison illusions—are just what we should expect if the visual system engages in latency correction. We have not, however, touched upon the Double Judd, the Müller-Lyer, or the Upside-down-T illusions. Each of these illusions involves the misperception of a projected distance or a projected size. Even the Hering illusion can be treated as a misperception of projected distance, since the projected distance between the two lines appears to be greater nearer the vanishing point. The Orbison illusion, too, can be classified as a misperception of projected size since the sides of the squares are not all perceived to be the same projected length. In this section we describe how latency correction explains these projected size illusions.

3.3.1 Projected x and y lines. How do the projected sizes of x and y lines change as an observer moves forward? Let us focus on how x projections change, and what we learn will immediately apply to y line projections as well. Figure 8a of Changizi (2001) demonstrates the manner in which a point in an observer's visual field moves horizontally away from the vertical meridian as the observer moves forward; it may also be understood by examining the projection sphere in figure 8a here. There is one major summary conclusion we can make concerning how projected x lines change as observers move forward:

The projected distance between any point and the vertical meridian increases as observers move forward. Furthermore, this projected distance increase is maximal for points lying along the horizontal meridian, and falls off as the point gets farther away from the horizontal meridian.



**Figure 8.** Projection spheres with x, y, and z line projections. (a) This aids us in understanding how projected sizes of x line projections change as an observer moves forward. The innermost pair of squares and circles depict the sides of a doorway that is far in front of an observer, the squares are at eye level (ie lying on the horizontal meridian) and the circles above eye level. The projected distance between the two squares is about the same as that between the two circles. But as an observer moves forward, in the next moment the sides of the door expand, the sides at eye level project as the next-farther-out pair of squares, and the sides above eye level project as the next-farther-out pair of circles. The horizontal projected distance between the squares is now greater than that between the circles. (The horizontal projected distance between the circles is the length of the great circle contour, a projected x line, connecting them on the projection sphere.) Similarly, in the next moment the sides are depicted by the next pair of squares and circles. (b) Identical to (a) but shows how vertical projected distances grow most quickly when they lie along the vertical meridian.

This statement is just another way of saying that as you approach a doorway, its sides bow out most quickly at eye level (and less and less quickly the further it is from eye level). The analogous conclusion holds for y lines:

The projected distance between any point and the horizontal meridian increases as observers move forward. Furthermore, this projected distance increase is maximal for points lying along the vertical meridian, and falls off as the point gets farther away from the vertical meridian.

These conclusions are sufficient to explain the projected size illusions shown in figure 4, except for the Upside-down-T illusion (which we take up in the next section). We will explain each in turn.

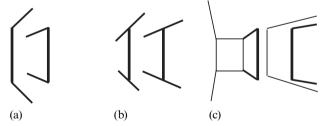
Double Judd. The Double Judd illusion consists of two projected y line segments, projections that do not cross the horizontal meridian (see figure 4). It suffices to treat each segment as if it were a point. We are interested in the projected distance between each segment and the horizontal meridian. They are, in fact, the same in the figure. However, the conclusion above states that in the next moment the segment nearer to the vertical meridian—ie the inner segment in figure 5—will have a greater distance from the horizontal meridian than the other segment. The latency-correction hypothesis therefore predicts that observers will perceive the segment that is nearer to the vertical meridian to have greater projected separation from the horizontal meridian. And this is just the illusion that occurs with the Double Judd stimulus: If the focus of expansion is up and to the right of the figure, then the right y line segment is nearer to the vertical meridian, and should be perceived to be lower (ie farther from the horizontal meridian which is above) than the left y line. Alternatively, if the focus of expansion is down and to the left of the figure, then the left y line segment is nearer to the vertical meridian, and should be perceived to be higher (ie farther from the horizontal meridian which is now below) than the right y line. [A similar explanation would work if the Double Judd stimulus was rotated by 90°.]

Müller-Lyer. The Müller-Lyer illusion consists of two projected y line segments, projections that do cross the horizontal meridian. Consider just the tops of each projected y line. The top of the projected y line on the left in figure 4 is nearer to the vertical meridian than the top of the other projected y line, and so it will move upward more quickly in the next moment. Thus, the projected distance between the top of the left projected y line and the horizontal meridian should appear to observers to be greater than that for the right projected y line. The same also holds for the lower halves of each projected line, and thus the total projected distance from the top to the bottom of the left projected line will be greater in the next moment than that of the right projected line, and thus should be perceived in that way if latency correction applies. And, of course, this is the illusion in the case of the Müller-Lyer stimulus.

*Ponzo*. The explanation for the Ponzo illusion follows immediately from the argument for the Müller-Lyer illusion, except that it concerns the distance from points to the vertical meridian.

Hering. In the Hering illusion in figure 4, there are two projected y lines on either side of the vertical meridian. The projected distance between the lines depends on how high one is looking above or below the horizontal meridian. At the horizontal meridian the perceived projected distance between the two projected y lines is greatest, and it falls as one looks up or down. The conclusion concerning x lines above explains this: points on one of the Hering lines nearer to the horizontal meridian will, in the next moment, move away from the vertical meridian more quickly. [A similar explanation would hold if the Hering stimulus had been presented as two projected x lines lying on either side of the horizontal meridian.]

We see, then, that one simple latency-correction rule underlies these four, seemingly distinct, classical geometrical illusions. The same explanation holds for any stimulus for which the probable sources and the probable focus of expansion are as above; and we may accordingly predict novel illusions. Figure 9 shows three such novel stimuli, and, as predicted by the model, each of these stimuli lead to the same kind of illusion (the last of which is contrary to the classical Müller-Lyer illusion).



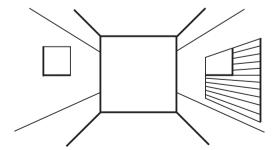
**Figure 9.** Three predicted illusions. The projected size of the vertical bold lines are the same in each figure, but the left one appears larger in each case because the cues suggest that the focus of expansion is to the left, and thus the left one will grow more quickly in the next moment. Note that in (c) the illusion is the opposite of the standard Müller-Lyer: the fins-in line appears longer than the fins-out line. This is because the entirety of cues suggests the focus of expansion is nearer to the left vertical line.

3.3.2 Projected z lines. The projected size and distance illusions discussed above concerned the projected sizes for x and y lines. What about the projected size of z lines? Consider how projected z line segments change as an observer moves forward. When the segment is very far away, it projects a small image, and as you get closer it projects a larger image. This is no different from the behavior of x and y lines. Consider, though, how a z line projection changes when you are already relatively nearby. It still projects larger in the next moment. This is partly because it is closer, but also partly because it begins to project more perpendicularly toward the observer.

Consider, as a contrast, how an x line segment lying on the horizontal meridian and to one side of the vertical meridian projects as an observer near it moves forward. Eventually, the x line begins to project less perpendicularly toward the observer—ie less of the line is facing the observer. When the observer passes the x line, its projected size will have fallen to zero. For the z line segment, however, when the observer passes it, its projected size will be at its maximum.

We can now ask and answer the question of how the probable source of the Upside-down-T illusion will change in the next moment. Recall that the source of the T is a corner made of x, y, and z lines, whose point lies on the horizontal meridian, and thus so do the x and z lines. The probable focus of expansion is somewhere on the same side as the z arm, but past the tip of the projected z arm (eg see figure 4). The projected size of the horizontal bar is due to the sum of the projected sizes of the x line and the z line, these lines being at right angles to one another in the world. Suppose each line has a length L m. Its projected size could then be mimicked by a single straight real-world line (it is not a principal line) going from the tips of each line, and whose real-world length is  $(L^2 + L^2)^{1/2}$ , that is 1.414L. The y line must, then, be approximately 1.414L m long as well, since it projects the same projected size and is approximately the same distance away. Consider now what happens when the observer is about to pass the corner. Since the xline is to one side of the vertical meridian, its projected size has fallen to  $0^{\circ}$ . The projected size of the z arm is at its maximum, however. The bottom of the varm rests on the horizontal meridian, and it will therefore not get smaller in the last moments before it is passed, but, instead, will increase to its maximum. Since the z line is of length L and the y arm length 1.414L, and since each is about the same distance from the observer, the projected size of the yarm will be about 1.41 times as large as the projected size of the z arm. This is how the corner will project when the observer is just passing it, but the more general conclusion is, then, that the total projected size of the bottom of the T grows less quickly than does the projected size of the y line. Latency correction therefore predicts that observers will perceive the vertical line to have greater projected size, as is the case.

In the explanation of the Upside-down-T illusion, we learned that, when relatively nearby, x line segments lying on the horizontal meridian and on one side of the vertical meridian—like the one in the Upside-down-T illusion—increase their projected size more slowly than do z line segments lying in the same part of the visual field. We can use this observation to predict a novel illusion. Figure 10 shows two identical



**Figure 10.** Two predicted illusions. First, the left horizontal line appears to have smaller projected size than the right one, but they are identical. The reason is that the right one is probably due to a z line (being part of the flag on the wall), whose projected size will increase in the next moment more than that of the x line on the left. Second, and for the same reason, the horizontal line on the right appears to have greater projected size than the adjacent vertical line, but the two lines on the left appear roughly identical (and, the predicted perception on the left is that the vertical line should be a little larger than the horizontal line).

horizontal lines lying on the horizontal meridian, one on each side of the vertical meridian. The one on the left has cues suggesting it is due to an x line, and the one on the right has cues to suggest that it is due to a z line. Although they are at equal distances from the vertical meridian, the z line appears to have greater projected size, as latency correction predicts. (The bold vertical lines are also identical in projected size to the horizontal lines.)

### 4 Psychophysical prediction and confirmation

Figure 11 shows how much a point in an observer's visual field moves away from the horizontal meridian in the next moment. The figure for movement away from the vertical meridian is identical, but rotated by  $90^{\circ}$ . This figure encapsulates and greatly extends most of the predictions and explanations the model of latency correction has made in this paper (namely, all those illusions that did not rely on misperception of the projected size of z lines).

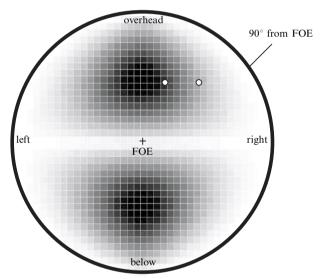


Figure 11. Change in projected distance from the horizontal meridian as a function of position within the visual field. The rim of the circle is 90 deg from the focus of expansion. The plot has a linear gray scale, with white representing 0 deg projected distance change, and black representing approximately 2 deg. The two dots are props referred to in the text. To help explain the plot, suppose you are walking down a long corridor toward a doorway at the end. When you begin, the top of the doorway is nearly at the focus of expansion (FOE), but just slightly above it. In the plot, the gray scale is very white here, telling us that the top of the door does not move upward in the visual field very much in the next moment. As long as you are far away, the top of the doorway moves slowly upwards in your visual field. As you get nearer, though, the doorway is high enough in your visual field that it is well within the darker regions above the focus of expansion in the plot, and it thus moves upward in your visual field very quickly in the next moment. As you begin to pass it, the doorway is nearly overhead, and slows down a bit before it finally goes behind you. The plot was generated by simulating forward movement around a point, starting at a z distance of 1 m, and moving with speed 1 m s<sup>-1</sup> for 100 ms. By rotating the plot by 90°, one obtains the plot for the change in projected distance from the vertical meridian as a function of position within the visual field.

We carried out experiments to test these predictions. With a computer, two dots were placed on a radial display of black lines, and successively moved as a pair to each of 300 different positions. For each position, the observer was asked to move the outer dot up or down until its perceived projected distance from the horizontal meridian was the same as that for the less-peripheral dot (see legend of figure 12 for methods).

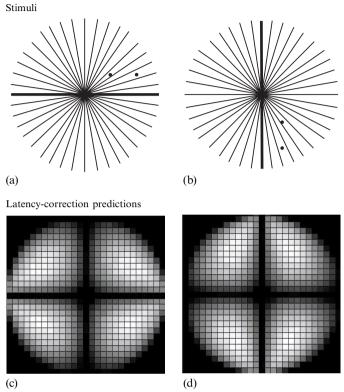


Figure 12. (a) A stimulus similar to the one used in the experiments measuring perceived projected distance from the horizontal meridian. The midpoint of the pair of dots was moved to 300 different positions in the radial display (an 18 by 18 grid, minus 6 points in each of the four corners which fell outside of the radial display) (the programming was done in Visual Basic). The arrows here indicate that observers were able to manipulate the height of the more peripheral dot by using a mouse to click up and down arrows; when the dots surrounded the meridian symmetrically, one dot was arbitrarily chosen to be the movable one. The actual stimulus had 64 black radial lines; the horizontal meridian line was bold to make it easier for observers to make their perceptual judgment of projected distance to the horizontal meridian. The two test dots were small red squares, and had a diameter of 2 mm, and they were (for each position in the display) initially set at the same projected distance from the horizontal meridian. The radial display had a diameter on the computer screen of 23 cm, the distance between the dots was 2 cm, and observers freely viewed the screen from approximately 60 cm away. The smallest allowable movement of a dot was 0.324 mm. (b) Same as (a) but for the purpose of measuring perceived projected distance from the vertical meridian. (c) The predicted misperception (for the latency-correction hypothesis) of projected distance from the horizontal meridian as a function of position in the visual field with respect to the focus of expansion. It is computed from the plot in figure 11 by computing the difference between two nearby dots in the amount of nextmoment change in projected distance from the horizontal meridian. White here means maximal predicted illusion, black means zero predicted illusion. The actual magnitude of the predicted illusion depends on the distance of the dots from the observer, and on their linear separation (as well as on the observer speed and latency); for reasonable values of these parameters the illusion magnitude is on the order of magnitude of a few deg (Changizi 2001), but here we will only be interested in comparing the first-order shapes of the predicted plots and the experimental plots. (d) Same as (c) but for the stimulus in (b).

The same experiment was run where the task was to judge the projected distance from the vertical meridian.

Before presenting the experimental results, let us examine exactly what the theory predicts. The data from observers in the above-mentioned experimental design are not of a form directly predicted by the plot in figure 11 because the observers are judging

the difference in projected distance from the horizontal meridian, whereas the plot measures how much any given point will move upward in the next moment. Instead, the predictive plot we want is the one that records, for each point in the visual field, how much more the less-peripheral dot will move away from the horizontal meridian than the more-peripheral dot. This plot can be obtained from figure 11 by simply taking, for each point in the visual field, the next-moment projected distance of the less-peripheral dot minus the next-moment projected distance of the more-peripheral dot. This is shown in figure 12; this figure shows the predicted strength of the vertical projected distance illusion as a function of position in the visual field. This one plot encapsulates the predicted illusion magnitude for nearly all the illusions discussed in this paper, as well as making many new predictions. If the visual system follows a latency-correction strategy, then we expect it to approximate the predicted plot, at least to first order; this plot is the fingerprint of latency correction. The predicted plot assumes that all points are equidistant from the observer, whereas in reality it may be that points at different positions in the visual field with respect to the focus of expansion have different probable distances from the observer. However, the basic qualitative features of the predicted plot are expected to be followed.

What are the basic features of the large-scale shape of the predicted plots? (1) The first principal feature is that there is a single predicted peak in each quadrant. A priori, this need not have been the case; there could, say, have been no illusion at 45 deg, and two maxima per quadrant, or negative illusions in some places, and so on. (2) The second main feature is that the predicted illusion magnitudes tend to be clumped nearer to the meridian from which observers are judging projected distance. This, too, need not have been the case. (3) The third general prediction we make is not explicitly represented in figure 12, but can be reasonably expected. The focus of expansion usually lies on the horizon, and there are more objects close to us—and thus moving more quickly in the visual field—below the horizon than above it, especially outdoors. We therefore expect that the probable distance from the observer is lower for dots below the focus of expansion, and that the illusion magnitude should be greater in the bottom half of the radial display, for either stimulus. We wish to compare the experimental results with these three predictions.

Two non-naïve observers (the two authors) and three naïve observers were tested on the stimuli shown in figure 12. Figure 13 shows the results for each individual on the horizontal-meridian and vertical-meridian versions of the experiment, and figure 14 shows the results averaged across the naïve observers, and averaged across the non-naïve observers. Qualitative features (1) through (3) may be readily seen in the experimental results: (1) there is single-peaked hump in each quadrant, (2) there is a tendency for the illusion magnitude to cluster toward the meridian from which observers judged projected distance, and (3) in 9 of 10 cases, observers perceived greater illusion magnitudes in the lower half of the radial display (see figure 13 for details).

We finish this section by discussing the appropriate manner in which dynamic stimuli may be used to test the latency-correction hypothesis. In one possible experiment that naturally springs to mind the observer would be in (real or simulated) motion in the vicinity of, say, the Ponzo illusion, and one might expect that the illusion magnitude would predictably vary depending on the observer's motion. This is, however, an inappropriate test of the hypothesis, as we now explain.

First, any such dynamic-stimulus experiment would have to constrain the kinds of motion to ecologically natural ones. It is plausible to expect that the visual system uses rough-and-ready rules to determine its percept. If in nearly all of our experiences with Ponzo stimuli we are in forward motion toward the vanishing point of the two obliques (ie if the probable focus of expansion really is the vanishing point of the obliques as the model entails), then it would be a reasonable strategy for the visual system to

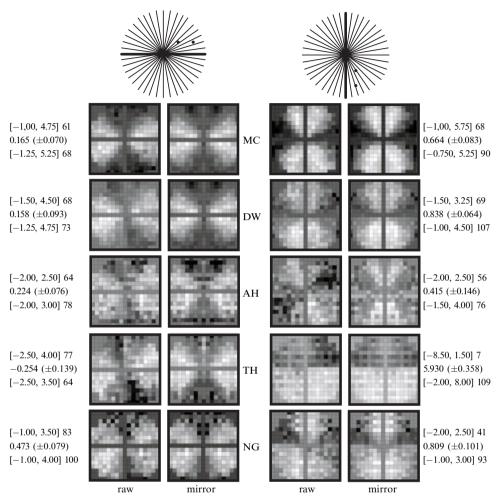
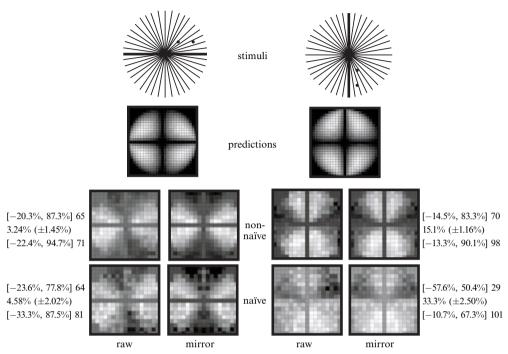


Figure 13. Experimental results for the two non-naïve (MC and DW) and three naïve observers on the two versions of the experiment. The first column under each stimulus shows the 'raw' averages for each subject for that stimulus (the non-naïve plots are averages of four experiments, and the naïve plots are averages over two experiments). Because nearly any a priori theory will make symmetrical predictions for the left and right halves of the visual field, we thought it reasonable to treat the left and right halves as replications of the same experiment. Accordingly, the second column under each stimulus is the same as the first, but where the results for the left and right halves of the visual field have been averaged together (labeled 'mirror'). Zero illusion is represented by whatever the gray level of the meridians are. To the sides of each experimental result are shown, for each of the upper and lower halves of the radial display, (i) the range of illusion magnitudes for the raw results (measured in clicks, and displayed as '[min, max]'), and (ii) the number of bins with a positive illusion (counted as positive if the bin value is greater than 10% of the maximum in the entire plot). Also shown in between the values for the upper and lower halves is (iii) the average difference (in number of clicks) between a bin in the lower half with its symmetrical bin in the upper half (with standard error in parentheses), positive values meaning a greater illusion in the lower half.

generate a percept guided by this, even if 1% of the time this fails to be an accurate representation owing to relatively infrequent observer motion. That is, there may be no need—and no ability even—for the visual system to be able to perceive the present for all possible observer motions. We should expect the efforts to be concentrated on the usual observer motions. Furthermore, there are kinds of observer motion that, even if they were common, do not as crucially need latency correction. For example, even if



**Figure 14.** Experimental results averaged over the non-naïve, and averaged over the naïve observers. Each observer's plot was normalized before averaging. As in figure 13, the first column under each stimulus shows the 'raw' averages, and the second column the 'mirror' averages. The numbers on the sides are as in figure 13, except that the range values and average difference value are measured not in number of clicks, but in percentages (since these plots are averages of normalized plots). The predicted plots are shown again for comparison.

moving backward were a common activity, perceiving the present is less important when moving backward since one is not about to run into the scene, and one can more easily afford to have a less than fully accurate percept.

A second problem with such an experiment is that the dynamic stimulus resulting from moving in the vicinity of a Ponzo (or any) figure is not ecologically appropriate, for it is not the same dynamic stimulus that would result from moving in the vicinity of the scene the figure depicts. A geometrical figure viewed statically causes a retinal stimulus, R<sub>0</sub>, that mimics the retinal stimulus that would emanate from some realworld source (or scene)  $S_0$ . The visual system is thus expected to react to that stimulus as if it is due to the probable real-world source (and that probable source may suggest the direction in which the observer typically moves in the next moment, and so on). Consider what happens, however, if the observer moves (in any direction). There will be some new retinal stimulus,  $R_1$ , but it will not be consistent with there being the same real-world source,  $S_0$ , as before. That is, how the figure projects as a function of distance from it will be radically different from how the real-world source,  $S_0$ , would project as a function of distance from it. Thus, the new retinal stimulus,  $R_1$ , will not be the stimulus that would occur had it been the case that the real-world source,  $S_0$ , had been doing the projecting onto the retina. The figure thus depicts a source,  $S_0$ , when the observer is at one position, but depicts a different source,  $S_1$ , when the observer is at the new position. In short, movement in the vicinity of a static geometrical figure is not consistent with the existence of a fixed real-world scene out there that the figure depicts. Although a single retinal stimulus from a figure is ecologically valid, the sequence of retinal stimuli caused by moving in the vicinity of the figure is not ecologically valid; rather, it produces a sequence of retinal stimulations that would never be encountered in the course of the observer's ecological experiences (outside of a psychology experiment). It is only reasonable to make predictions from ecologically valid stimuli—what should occur for the infinitely many possible nonecological stimuli psychologists may invent is entirely opaque.

One idea for avoiding the previous problem is to put the observer in motion, and to have the Ponzo stimulus briefly flashed, and to see if the perception is as predicted. Unfortunately, this proposal, too, has problems. The difficulty with this is that the probable motion (relative to the observer) of a source can reasonably be expected to depend on whether the stimulus is present the entire time the observer is moving, or whether the stimulus is just briefly flashed. It seems plausible that the probable observer-relative speed of the source of a briefly flashed stimulus is zero; ie without cues to the contrary, it may be that a flashed stimulus is assumed to be due to an object that is stationary relative to the observer, in which case no illusion would be expected. [We also note that simply replacing the radial lines in the Ponzo illusion with radially outflowing texture would not be an appropriate test either, since in that case the probable motion of the two horizontal lines relative to the observer is again zero (lest the lines also be flowing radially outward).]

What kinds of predictions can we make for the latency-correct hypothesis? One obvious prediction that is ecological is to see whether illusions vary as a function of the probable direction of observer motion. The psychophysical experiment we described earlier did just this. If one wishes to carry out latency-correction experiments with dynamic stimuli, one must either have the observer move in the vicinity of a real-world source, or, equivalently, have a geometrical figure on a computer screen that changes dynamically (so as to mimic a single ecological source changing in the manner it projects because of distance or position in the visual field). One of the simplest ways to carry this out on a computer would be to have a single dot moving radially outward from a focus of expansion, and to test whether observers employ latency correction in their perception of the dot's perceived projected distance from the (say) vertical meridian. When the dot is at some particular projected distance from the vertical meridian, another dot could be flashed, above or below the moving dot, and at the same time projected distance from the vertical meridian. Because the flashed dot has no cues that it is moving relative to the observer, and because the moving dot does have such cues, we expect observers to perceive the moving dot to have already achieved a greater projected distance from the vertical meridian than the flashed dot. That is, the prediction is that the flashed dot should lag behind the moving dot. But notice that this kind of dynamic experiment falls exactly within the class of flash-lag experiments, where something that is moving in a predictable fashion is perceived to lead something that is flashed. Thus, if one wishes to carry out latency-correction experiments with dynamic stimuli, one must engage in exactly the kinds of experiments that those like Nijhawan and colleagues already have been engaged in for some years now. In other words, the kinds of predictions made for dynamic stimuli have already been tested by others, and although there is a lively debate surrounding the class of flash-lag experiments, the existence of the effect is prima facie support for latency correction.

#### 5 Conclusion

We have argued that, owing to serious conceptual difficulties, the traditional inference approach cannot explain the illusions. The main problem is the supposed functionality of the approach: under the most charitable interpretation, the theory provides no good reason for the misperceptions of the projective properties.

We then fleshed out the consequences of the latency-correction hypothesis, and showed how a wide variety of classical geometrical illusions may be derived from it. We showed how each geometrical figure possesses cues concerning the kinds of real-world line segments it depicts, as well as cues as to the observer's probable direction of motion relative to the figure. And we found that the misperception in each case is consistent with the manner in which the real-world line segments would project in the next moment were the observer moving in that direction. We then used the hypothesis to predict several novel illusions, namely some variants of the Müller-Lyer illusion, and some of the Upside-down-T illusion. Finally, we examined a general class of (300) predicted illusions—each concerning the perceived relative positions of two dots in a radial display—and found that the illusion magnitude varies in a manner consistent with how the probable source would project in the next moment were the observer moving toward the center of the radial display.

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