Perception, 2001, volume 30, pages 195 – 208

DOI:10.1068/p3158

'Perceiving the present' as a framework for ecological explanations of the misperception of projected angle and angular size

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Received 12 April 2000, in revised form 30 October 2000

Abstract. An implicit, underlying assumption of most Helmholtzian/Bayesian approaches to perception is the hypothesis that the scene an observer perceives is the probable source of the proximal stimulus. There is, however, a nontrivial latency (on the order of 100 ms) between the time of a proximal stimulus and the time a visual percept is elicited. It seems plausible that it would be advantageous for an observer to have, at any time t, a percept representative of what is out there at that very time t, not a percept of the recent past. If this is so, it implies a modification to the implicit hypothesis underlying most existing probabilistic approaches to perception: the new hypothesis is that, given the proximal stimulus, the scene an observer perceives is *the probable scene present at the time of the percept*. That is, the hypothesis is that what an observer perceives is not the probable source of the proximal stimulus, but the probable way the probable source will be when the percept actually occurs. A model of an observer's typical movements in the world is developed, and it is shown that projected angles are perceived in a way consistent with the way the probable source will project to the eye after a small time period of forward movement by the observer. The predicted and actual direction of projected-angle misperception is sometimes toward 90° and sometimes away from 90°, depending on whether the probable source angle is lying in a plane parallel or perpendicular to the probable direction of motion, respectively. The perception of angular size for lines in a figure with cues they are lying in a plane perpendicular to the direction of motion is also shown to fit the predictions of the model.

1 Introduction

There is a latency on the order of magnitude of 100 ms (Lennie 1981; De Valois and De Valois 1991; Maunsell and Gibson 1992; Schmolesky et al 1998) between the time light hits the retina and the time a percept is elicited. To see that this latency is ecologically significant, consider that if an observer moving at 1 m s^{-1} were to perceive the world the way it was at the time light hit the retina, the distances to objects along the observer's direction of motion would be misperceived by 10 cm, and, in particular, any object perceived to be within 10 cm of passing the observer would already be behind him by the time he perceives it. It therefore seems intuitively clear that it is advantageous for an observer to have at any time t a percept representative of the scene present at time t, rather than a percept representative of the recent past (De Valois and De Valois 1991; Nijhawan 1994, 1997). The main result of this paper is that, within this 'perceiving the present' framework, an explanation can be given for the long-known phenomenon that observers perceptually overestimate acute ambiguous projected angles and underestimate obtuse ambiguous projected angles. [*Perceived projected angle* refers to the perception of the angle projected by a 3-D source angle toward an observer. For example, the corner of the ceiling above you is probably 90° (this is the 3-D source angle), but it is probably currently projecting to your eye at between 120° and 160°.] In particular, a simple model is put forth describing an observer's typical movements through a carpentered world, and it is shown that projected angles are perceived in accordance with the way the probable 3-D source angle probably will project in the

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next moment, ie at the time the percept actually occurs. The model not only explains why ambiguous projected angles are perceived nearer to 90° than they are, but it successfully predicts that projected angles with cues that the angle is lying perpendicular to the direction of motion are perceived *away* from 90°. In addition, the model is successfully applied to the perception of the angular size of objects with cues they are lying perpendicular to the direction of motion.

2 Model of simplified movement through a simplified world

If the visual system does engage in a 'perceiving the present' strategy, then, in addition to facing the standardly recognized ambiguity that there are multiple possible scenes that could have caused the proximal stimulus, it faces a second kind of ambiguity: that, for any scene consistent with the proximal stimulus, there are multiple possible ways in which the scene may change in the next moment. It would seem a good starting point to assume that, for any proximal stimulus, what principally drives perception under a 'perceiving the present' strategy is the empirically most probable way the empirically most probable scene changes in the next moment. If a proximal stimulus is predominantly received under conditions where the scene changes in a systematic fashion, then the 'perceiving the present' hypothesis predicts that the visual system will elicit a percept on the basis of this systematic change—even when the proximal stimulus is presented under the rarer conditions where it is not undergoing the systematic change [this would be 'inappropriate' (Gregory 1963) perceiving the present].

My hypothesis is that the geometrical illusions are cases of inappropriate perceiving the present: the proximal stimuli from geometrical figures (such as an angle on a piece of paper) are predominantly received from 3-D contours in scenes during an observer's movement through the world, and the visual system elicits a percept based on the systematic changes predominantly experienced. Note that on this view we would expect that greater exposure to a (static) geometrical figure will increase the probability that the scene will not change in the next moment (ie increase the probability that the scene is actually static), and thus the illusion should be reduced; this is in agreement with experiments (eg Brosvic et al 1997; Predebon 1998). In order to give a 'perceiving the present' explanation why a particular proximal stimulus leads to a particular perception, we need to be able to answer two questions: (i) For any proximal stimulus, what is the empirically most probable scene consistent with it? (ii) For any scene, what is the empirically most probable way in which an observer's view of it changes in the next moment (ie by the time the percept is elicited)? Answering these questions can generally be very difficult. For geometrical proximal stimuli of the kind examined here, however, there are some plausible, simplifying assumptions that can aid us in answering them.

The experiences of human observers are more often in carpentered environments than in any other; people move down roads and walk through rooms and hallways. [People raised in non-carpentered environments experience little or no illusion in the geometrical proximal stimuli (Gregory 1997, pages 150 – 151).] In such environments, there are primarily three *principal axes*: a y axis perpendicular to the ground plane (eg the boundary between two walls), and two mutually perpendicular x and z axes parallel to the ground plane (eg the boundaries between walls and the floor). Lines parallel to these principal axes are *principal lines*, and specifically are either *x lines*, *y lines*, or *z lines*, respectively. Given that principal lines are the most commonly encountered kinds of line in people's experience, it is reasonable for us to assume that, unless there are cues to the contrary, any projected line in the proximal stimulus is probably caused by either an x, y, or z line.

Furthermore, consider how people typically move through carpentered scenes. First, most movement is parallel to the ground plane; climbing stairs and ladders is relatively uncommon. Second, people tend to move down hallways, roads, and paths so that one of the principal axes parallel to the ground plane is perpendicular to the direction of movement (eg the cracks in the sidewalk) and the other is parallel to the direction of movement (eg the sides of the sidewalk). I will assume from here on that x lines lie perpendicular to the direction of motion and z lines lie parallel to the direction of motion. Figure 1 depicts an example such scene. Note that, since motion is assumed to be parallel to the z axis (and is assumed to be forward movement), the vanishing point of the projected z lines is also the focus of expansion. This kind of movement will be referred to as *orthogonal movement*.

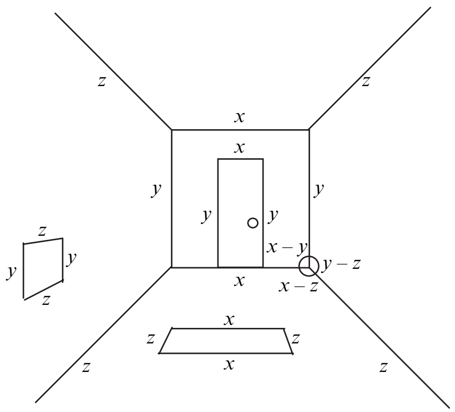


Figure 1. Examples of the three kinds of principal lines: x (parallel to the ground plane and perpendicular to the observer's direction of motion), y (perpendicular to the ground plane), and z (parallel to the ground plane and to the observer's direction of motion). Horizontal projected lines usually indicate projections of x lines, vertical projected lines usually indicate projections of y lines, and oblique projected lines usually indicate projections of z lines. Also shown are examples of the three kinds of principal angle: $x-y$ (built with x and y lines), $x-z$ (built with x and z lines), and $y-z$ (built with y and z lines).

We can categorize all the possible kinds of projected line that may come from principal lines when under orthogonal movement. When a line in the world projects to the eye, its projection may be characterized as a contour on an imaginary *projection sphere* with the viewer (or cyclopean eye) at its center. For any object in the world, the angle of subtense at the eye may be expressed by projection to the surface of this projection sphere. Figure 2 shows three projection spheres; in each there is a cross marking the position in the visual field of the focus of expansion. The solid curves in the left viewing sphere show how x lines project when they are perpendicular to an observer's direction of motion (as is assumed above to be most probable). Each of these x line projections is defined to be a *visually horizontal line* in the visual field. The solid curves in the middle viewing sphere show how y lines project when they are perpendicular to an observer's direction of motion (as is assumed above to be most probable since people walk along the ground plane). Each of these projections of y lines is defined to be a *visually vertical line* in the visual field. Projections of z lines are shown in the right viewing sphere. Projections of z lines are almost always *visually oblique*, meaning that the projection onto the viewing sphere is neither visually horizontal nor visually vertical.

Most of a human observer's viewing time is spent viewing objects along a horizontal band of the viewing sphere, from the left side, through the focus of expansion, and to the right side, and this can help us decide what kind of principal line is probably the source of a projected line. The region bound by the dotted ellipse in figure 2 is roughly this region. Given that people view out of the horizontal band more often than any other viewing region, it is reasonable to assume that, unless there are cues to the contrary, projected lines in a geometrical figure are interpreted by the visual system to be due to projections on the horizontal band of the viewing sphere. Consider how lines project within the horizontal band, as can be seen in figure 2. First, note that y lines typically project nearly parallel to one another and extend from the top straight down to the bottom; x lines also typically project parallel to one another in the horizontal band, but extend from the left side straight to the right side. However, in the left and right

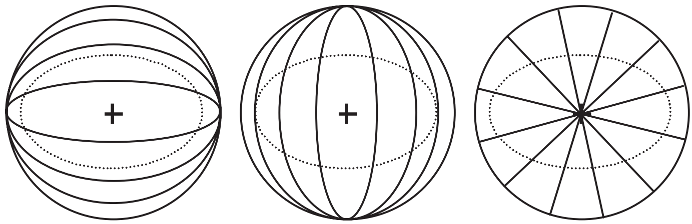


Figure 2. A viewing sphere represents all possible views from a given location. The viewer is at a point at the center of a viewing sphere. Three viewing spheres are shown. In each, the cross indicates the focus of expansion, ie the direction of movement. All curves and lines depicted in the figure are on the near surface of the spheres. On the left viewing sphere, the solid curves show how x lines (lines parallel to the ground and perpendicular to the observer's direction of motion) of infinite extent project to the eye. For example, if an observer is standing in front of a railroad crossing, the rails are x lines, and they project to the observer's eye below the focus of expansion; the parts of the rail far off on either side project onto the more peripheral parts of the sphere. Each of these x line projections is, by definition, a *visually horizontal line*. On the middle viewing sphere, the solid curves show how y lines (lines perpendicular to the ground) of infinite extent project to the eye. Each of these y line projections is, by definition, a *visually vertical line*. On the right viewing sphere the solid curves show how z lines (lines parallel to the ground and parallel to the observer's direction of motion) of infinite extent project to the eye. For example, the line marking the side of the road heads off in front of an observer, and it projects a line below the vanishing point, eventually approaching the vanishing point as an observer looks off into the distance. The region bound by the dotted curve is the *horizontal band*, and roughly marks the region observers most commonly view (see text).

peripheral regions of the horizontal band, x lines can begin to show some degree of non-parallelism, bending in toward one another (sharing a vanishing point on a peripheral pole of the sphere). Finally, within the horizontal band, z line projections are not parallel to one another; they typically cross diagonally, and they share a common vanishing point.

From these observations we can conclude the following rules. (i) Parallel lines in a figure that extend from the top straight down to the bottom of the figure are most probably due to y lines. (ii) Parallel lines in a figure that extend from the left side straight to the right side of the figure are most probably due to x lines. (iii) If there is a set of diagonal lines in the figure which are pairwise non-parallel and share a common vanishing point, then they are most probably due to z lines. (iv) If there are two sets of diagonal lines, each set with its own distinct vanishing point, then the lines in one set are probably due to z lines, and those in the other set are due to x lines; ie the view is in a peripheral region of the horizontal band. These rules are of no help when there are contours in the proximal stimulus inconsistent with the assumptions (eg curved contours or more than two vanishing points). Accordingly, I confine myself to studying geometrical proximal stimuli that are capable of being treated in the manner discussed above. [Note that only (i), (ii), and (iii) are required for the geometrical proximal stimuli encountered in this paper.]

As an example, consider how we may use these rules to determine which kind of principal line causes each projected line in figure 1. First, note that when an observer views figure 1 his/her impression is that the view is parallel to the ground, and that the kind of principal line causing any given projected line is as shown in the figure. However, it is possible that the figure is actually depicting a view directly up at a high ceiling with a strange overhead door; if this were so, the diagonal lines would be due to y lines, not z lines. From the rules, though, we can conclude that this latter possibility is not probable: from rule (i) we can conclude that the vertical lines in figure 1 are due to y lines, from rule (ii) we can conclude that the horizontal lines are due to x lines, and from rule (iii) we can conclude that the oblique lines are due to z lines.

With the assumptions discussed above, it is possible for us to determine which projected lines in the proximal stimulus are x lines, which are y lines, and which are z lines. In doing so, we thereby determine the most probable scene given the proximal stimulus, at least at the level of detail with which we will be concerned. The 'perceiving the present' hypothesis predicts that the geometry an observer perceives is representative of the way the most probable scene most often is in the next moment (ie at the time of the percept). To figure out how the scene will be in the next moment, we can simply figure out how the principal lines in the scene project after an observer moves a certain amount in the z direction toward the vanishing point. Observers move at a large variety of speeds, and to estimate the empirically most probable way in which an observer's view of a carpentered scene changes in the next moment, we must estimate the empirically most probable speed at which people travel in carpentered environments: throughout this paper I use the conservative estimate of 1 m s^{-1}. I will assume a conservative proximal-stimulus-to-perception latency of 50 ms.

One line of evidence for 'perceiving the present' is from MacKay (1958), Nijhawan (1994, 1997), and Schlag et al (2000) [see also Sheth et al (2000)], who have shown that, when a stationary object is flashed in line with a continuously moving object, the flashed object is perceived to lag behind the moving object, the lag corresponding to a hypothetical latency of around 80 ms (Nijhawan 1994); although this 'perceiving the present' interpretation is debated (Baldo and Klein 1995; Khurana and Nijhawan 1995; Lappe and Krekelberg 1998; Purushothaman et al 1998; Whitney and Murakami 1998; Krekelberg and Lappe 1999; Brenner and Smeets 2000; Eagleman and Sejnowski 2000; Khurana et al 2000; Whitney et al 2000). Some of this extrapolation may even be carried out by retinal ganglion cells (Berry et al 1999). Existing evidence for inappropriate perceiving the present can be found in Thorson et al (1969), who have shown that when two very nearby points are consecutively flashed, motion is perceived to extend beyond the second flashed point. Another line of evidence for inappropriate perceiving the present can be found in Anstis (1989) and De Valois and De Valois (1991), who have shown that stationary, boundaryless figures with internal texture moving in a direction induce a perceived figure that is substantially displaced in the same direction (see also Nishida and Johnston 1999; and Whitney and Cavanagh 2000), the displacement corresponding to a hypothetical latency of around 175 ms (Anstis 1989).

3 Perception of ambiguous projected angles and projected angles with cues they lie in a plane parallel to the direction of motion

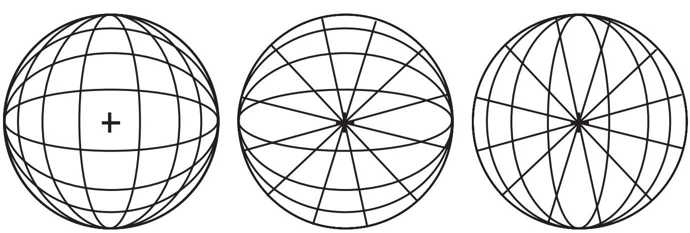
It has long been known that, when an observer is presented with a projected angle with poor cues as to the 3-D source angle (even a simple line drawing, such as '$<$', suffices), the observer misperceives the projected angle, tending to perceive it more toward 90° than it actually is (Fisher 1969; Bouma and Andriessen 1970; Carpenter and Blakemore 1973; Nundy et al 2000). The maximum absolute magnitude of the misperception is around 2° or 3°, the misperception magnitude falling to zero at 0°, 90°, and 180°.

Although Carpenter and Blakemore (1973) suggest a possible mechanism for the misperception (namely, lateral inhibition), we are here interested in considering ecological explanations. The main historical ecological explanation for projected angle is constancy-scaling (Gregory 1963, 1997; Gillam 1980). The constancy-scaling argument might proceed something like this: (#) a 30° projected angle is probably due to a 90° 3-D angle (for ecological reasons), and the perception of angle deviates from the projection and towards the 3-D angle, leading to a perception of, say, a 32° angle. Stated in this manner, one gets the impression that a person can have only one kind of angle perception, and that by generating a percept of an angle that is pushed toward 90°, one gets a more veridical percept (since the angle perception is nearer to the source angle). But this is mistaken. This seeming function of constancy-scaling is attractive only when one conflates two distinct kinds of angle perception: the perception of projected angle (perception of the angle projected by the 3-D source angle toward the observer, which depends on both the 3-D source angle subtense and its orientation with respect to the observer) and the perception of 3-D angle (perception of the subtense of the 3-D angle, irrespective of the orientation of the 3-D angle with respect to the observer). Just as we have simultaneous perceptions of roundness and redness, and of (more analogously) brightness and lightness, we also have simultaneous perceptions of projected angle and 3-D angle. For example, concerning an upper corner of the room you are in, you (i) perceive three obtuse projected angles, and at the same time you also (ii) perceive three 90° 3-D angles; if you move about, your first percept will continuously change, while your second will stay constant. By conflating these two notions, constancy-scaling can appear to have a nice ring to it. But, if we restate (#) from above, but now employing both notions of perceived angle, we instead get: a 30° projected angle is probably due to a 90° 3-D angle (for ecological reasons), and the perception of the *projected* angle deviates from the actual projection and towards the 3-D angle, leading to a perception of, say, a 32° *projected* angle. Now, this no longer seems functionally useful at all. Prima facie, to perceive veridically would be (i) to perceive the projected angle to be 30°, and (ii) to perceive the 3-D angle to be 90°. What could be functionally useful about perceiving a 30° projected angle to have a 32° projected angle? As far as I understand it, constancy-scaling provides no answer. Constancy-scaling thus leaves us in the dark why projected angles are misperceived. A recent, ecological explanation has been proposed by Nundy et al (2000) that has many similarities to constancy-scaling. They argue that a 30° projected angle is more probably caused by a 3-D source angle with a subtense greater than 30° since, intuitively, there are more 3-D source angles greater than 30° that can project as 30° than there are 3-D source angles less than 30° that can project as 30°. That is, they argue that the probable source angle of a 30° projected angle is a 3-D source angle with subtense greater than 30°. Within a probabilistic framework for visual perception, where it is hypothesized that observers perceive the probable source of the proximal stimulus, they conclude that observers overestimate the 30° projected angle because the 3-D source angle is probably bigger. This argument, too, conflates perception of 3-D angle with the perception of projected angle. While it may be true that the probable 3-D source angle is probably bigger than the projected angle, the prediction following from this is that observers will perceive the 3-D source angle to be bigger than the projected angle. It does not follow that observers will perceive the projected angle to be bigger than it is, and the argument thus does not touch upon the phenomenon. In many places in the literature a similar conflation occurs between lightness and brightness, and also between surface color and color (see, eg, references cited in end-notes 11 and 12 of Arend and Goldstein 1990).

In applying the 'perceiving the present' framework to this problem, we first must gauge the probable source of the proximal stimulus, and then determine the probable way the probable source projects in the next moment. Under the assumption that all projected lines in proximal stimuli are projections of x, y, or z lines, it follows that every projected angle is due to one of only three kinds of 3-D angle, each a right angle: *$x-y$ angles*, angles formed from an x line and a y line; *$x-z$ angles*, angles formed from an x line and a z line; and *$y-z$ angles*, angles formed from a y line and a z line. Examples of these three kinds of *principal angle* are given in figure 1. Of these three kinds of principal angle, $x-z$ and $y-z$ are similar in that each lies in a plane parallel to the direction of motion; $x-y$ angles, on the other hand, lie in a plane perpendicular to the direction of motion. '$xy-z$ angles' will denote $x-z$ or $y-z$ angles. How the projection of a principal angle will change in the next moment depends on which kind of principal angle it is: $x-y$ or $xy-z$. There are two reasons to believe that, when there are little or no cues, projected angles are probably $xy-z$ angles.

First, $xy-z$ angles are probably more frequent in an observer's experience. Consider that of the three kinds of principal angle—$x-y$, $x-z$, and $y-z$—two are $xy-z$ angles. If we assume that the relative frequency of encountering principal angles is equally divided between $x-y$, $x-z$, and $y-z$ angles, then two thirds of the possible principal angles are $xy-z$ angles. That is, every corner consists of each of the three kinds of principal angle, and thus $xy-z$ angles are, among corners, twice as frequent as $x-y$ angles. For example, among the principal angles formed by intersections of the walls, ceiling, and floor in figure 1, there are 8 $xy-z$ angles (4 $x-z$ and 4 $y-z$) and 4 $x-y$ angles.

Second, not only are $xy-z$ angles probably more frequent in an observer's experience, they are also by far the primary source of projected angles that differ appreciably from 90°. Intuitively, this is because $x-y$ angles project to the eye at around 90° until an observer nearly passes by them, only at which time do they change rapidly away from 90°. To see this we may examine figure 3. The projected angles on the $x-y$ viewing sphere are very near 90° until very far from the focus of expansion. The projected angles on the $xy-z$ viewing spheres, however, come in a large variety of angles even when near the focus of expansion. These observations may be quantified by looking at a histogram of the number of times a principal angle projects a certain angle in a simulation of natural movement around an angle, for both $x-y$ and $xy-z$ principal angles. Figure 4 shows these two histograms, and one can see that $x-y$ principal angles usually project very near 90°, and that projections below around 75° or above around 105° are mostly caused by $xy-z$ principal angles. In short, most acute and obtuse angles are due to $xy-z$ principal angles.



$x-y$ angles $x-z$ angles $y-z$ angles

Figure 3. How the three kinds of principal angle project onto the viewing sphere as a function of position in the visual field. (Note that the angles actually printed on this page are not the projected angles. Rather, the figure depicts a 3-D viewing sphere, on which angles have been projected.) One can see that $x-y$ angles project as nearly 90° when they are near the focus of expansion, but project farther and farther from 90° as they get more distant from the focus of expansion. (To see this, one must judge the 3-D angles depicted in the viewing-sphere figure.) Since optic flow is from the focus of expansion outward, $x-y$ angles are predicted to be misperceived away from 90°. One can also see that $x-z$ angles and $y-z$ angles ($xy-z$ angles) typically project very differently than 90° when near the focus of expansion, but project more and more toward 90° as they get more distant on the viewing sphere from the focus of expansion. The prediction is, therefore, that $xy-z$ angles are misperceived toward 90°.

If 'perceiving the present' is influenced primarily by how the projection of the most probable source of a projected angle changes in the next moment, then we expect that it is the dynamics of $xy-z$ angles that drives the perception of ambiguous projected angles. Ambiguous projected angles should, then, be perceived in a manner similar to the way projected angles with cues that they are $xy-z$ angles are perceived. It has long been known that cue-rich projected $xy-z$ angles are misperceived, or 'regressed,' toward 90° (eg Thouless 1931). For example, a rectangle lying flat on the ground in front of an observer is perceived to project more like the 'real' rectangle it actually is than its actual projection. What is the perception predicted by the 'perceiving the

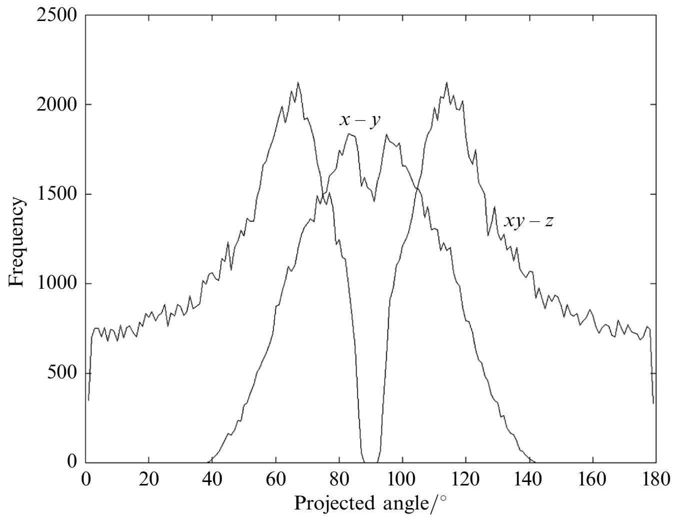


Figure 4. Histogram of counts for the projections of $x-y$ and $xy-z$ angles. One can see that $x-y$ angles rarely project angles much differently than 90°; most acute and obtuse projected angles are due to $xy-z$ angles. The curves were generated by simulating 10^5 movements near an angle of the specified kind (10^5 for each of $x-y$, $x-z$, and $y-z$). Each 'movement' consisted of the following. First, a random orientation of the principal angle was chosen. For example, for an $x-z$ angle there are four orientations: $+x$ and $+z$, $+x$ and $-z$,$-x$ and $+z$, and $-x$ and $-z$. Second, the angle's vertex was placed at the origin. Third, a pre-move viewing position for the simulated observer was determined by randomly choosing values for x uniformly between 0.1 m and 1 m to one side of the angle, values for y uniformly between 1 m above and below the angle, and values for z uniformly between 0.5 m and 1 m in front of the angle. The simulation was confined to these relatively nearby positions since one might expect that veridical perception of nearby objects matters more in survival than veridical perception of objects far away. The nature of my conclusions does not crucially depend on the particular values used in the simulation. Finally, the observer was moved along the z axis toward the angle (ie z got smaller) at a speed of 1 m s^{-1} for (a latency time of) 0.05 s.

present' hypothesis? Consider, for example, the farther-away $x-z$ angle on the right side of the 'rug' on the floor of the room depicted in figure 1. This angle has an obtuse projection in the figure, and as an observer walks forward, the projection will move toward 90°, reaching exactly 90° as the observer passes it. The nearer $x-z$ angle on the right side of the rug behaves similarly, except that it begins with an acute projection and moves toward 90°. Figure 5 shows how, more generally, the projections of $xy-z$ angles change in the next moment as a function of their projected angle (the inset shows sample data from the literature with which to compare it). One can see that acute projected angles get larger, and obtuse projected angles get smaller, with no change for projected angles of 0°, 90°, and 180°. Another way to understand how $xy-z$ angles change is to examine the middle and right viewing spheres in figure 3. Consider the projections of $y-z$ angles. When a $y-z$ angle is near the focus of expansion, its projected angle is typically either very acute or very obtuse. If we examine the $y-z$ projected angles along a radial line from the focus of expansion outward, we can see that the projected angles progressively approach 90°.

In sum, ambiguous projected angles differing from 90° are probably $xy-z$ angles, and $xy-z$ angles tend to project nearer to 90° in the next moment. Thus, acute ambiguous projected angles are predicted to be overestimated, and obtuse ones underestimated, which is in agreement with the data (figure 5, inset). Furthermore, we have seen that the predicted misperception goes to zero at 0°, 90°, and 180°, and that the maximum predicted misperception is the right order of magnitude (a few degrees or so); these are also in agreement with the data.

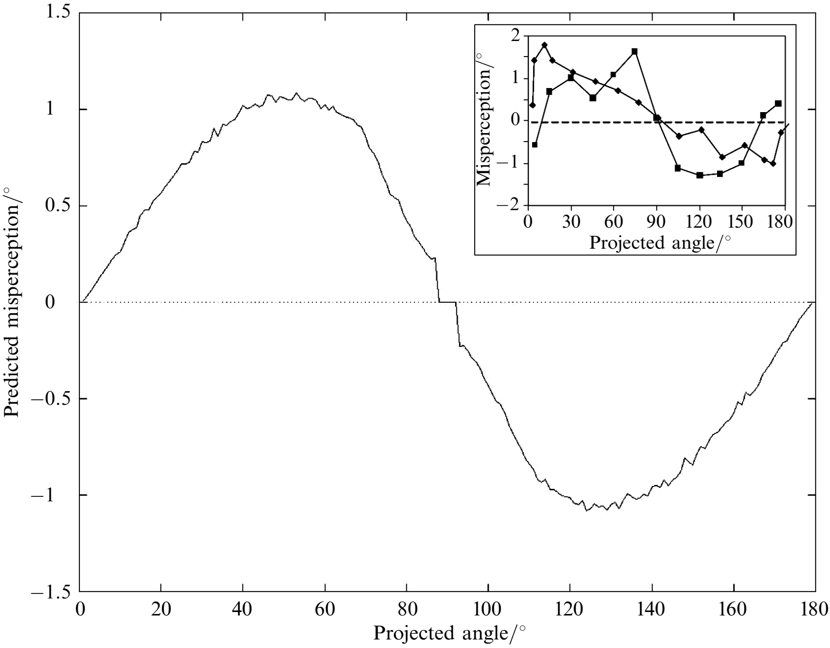


Figure 5. Average projected angle change as a function of the pre-move projected angle, for principal right angles lying in a plane parallel to the direction of motion ($xy-z$ angles). One can see that the 'perceiving the present' hypothesis predicts that, for projected angles that are probably due to $xy-z$ angles, acute projected angles are overestimated and obtuse projected angles are underestimated. The graph was generated from the same simulation as that described in figure 4. The particular position of the peak is not important, as it depends on the allowed range of pre-move positions in the simulation. Inset shows two plots of actual misperceptions for subjects. Diamonds are averages from one representative non-naïve subject (RHSC) from Carpenter and Blakemore (1973, figure 3), and squares are averages from six naïve subjects from Nundy et al (2000, figure 5).

4 Perceived projected angle for proximal stimuli with cues they are in a plane perpendicular to the direction of motion

We have examined how $xy-z$ angles change in time, but another test of the theory is to see if it successfully predicts the perception for projected angles with cues that the 3-D angle is an $x-y$ angle. When an $x-y$ angle is sufficiently far in front of an observer, it projects to the eye as nearly 90°. As an observer approaches and begins to pass it, however, it projects an angle either progressively greater or progressively smaller than 90°, depending on its orientation with respect to the observer. You can see this yourself by holding out a piece of paper in your frontoparallel plane and moving it past you. Another way of seeing this is to examine a viewing sphere on which x lines and y lines are projected, as in the left viewing sphere in figure 3. Every angle on this viewing sphere is the projection of some $x-y$ angle, and one can see that $x-y$ angle projections near the focus of expansion are near 90°, but $x-y$ angle projections away from the focus of expansion get progressively different from 90°. As an observer moves, objects in the visual field start from near the focus of expansion and move radially outward. Thus, to understand how the projections of $x-y$ angles change in the next moment, we can move radially outward on the viewing sphere figure from the focus of expansion and see how the projection changes. So, for example, consider the bottom left angle of the 'square' around the focus of expansion on the left viewing sphere in figure 3. It is roughly 90°. Now move to the square below and to the left of the first square, and consider its bottom left angle. This angle is along the same radial as the first angle, and is thus the way the first projected angle will change in time; in this case it becomes progressively more obtuse.

Which way the projection of an $x-y$ angle changes depends on its orientation with respect to the observer. Figure 6 shows how the projection of one particular $x-y$ angle—one with a $+x$ arm and a $+y$ arm—changes as a function of its vertex's position on a plane 1 m ahead of an observer. The reader can see that, when the angle is in either the first or third quadrant, the projected angle gets bigger in the next moment. Alternatively, when the angle is in either the second or fourth quadrant, the angle gets smaller in the next moment. Similar plots can be generated for $x-y$ angles with arms pointing in other directions. Figures 7a and 7c summarize the directions of change in surface plots like figure 6 for the angles in, respectively, four and nine squares around the focus of expansion. Figures 7b and 7d show the same squares as, respectively, in figures 7a and 7c, but in a proximal stimulus with strong cues suggesting that the vanishing point, and thus the direction of motion, is in the center, and that the angles are in a plane perpendicular to the direction of motion ($x-y$). (Cues for the position of the focus of expansion are crucial for misperception of the projection of $x-y$ angles because how the projection changes depends on where the focus of expansion is with respect to it. $xy-z$ angles change toward 90° irrespective of the position of the focus of expansion, and thus observers experience misperception of acute and obtuse projected angles even without cues as to the position of the focus of expansion.) As the reader may verify for himself/herself—and as was the case for the more than fifteen people to whom I have shown figures 7b and 7d—the actual directions and relative magnitudes of the misperceptions are in agreement with the trends in figure 6.

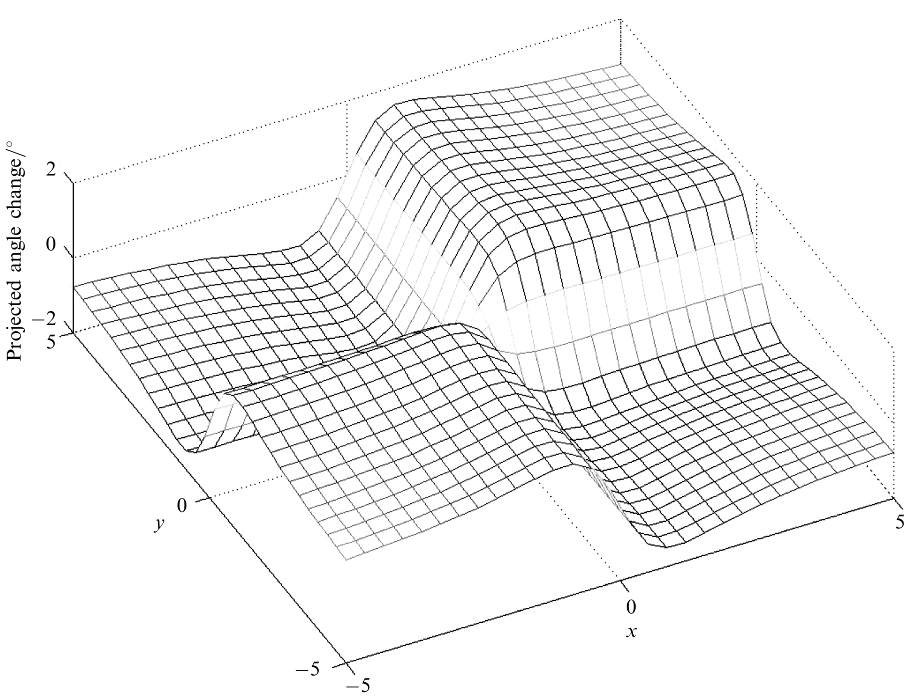


Figure 6. The change in projected angle as a function of position with respect to the focus of expansion on a plane 1 m ahead and perpendicular to the direction of motion, for an $x-y$ angle with one arm pointing up and another arm pointing right.

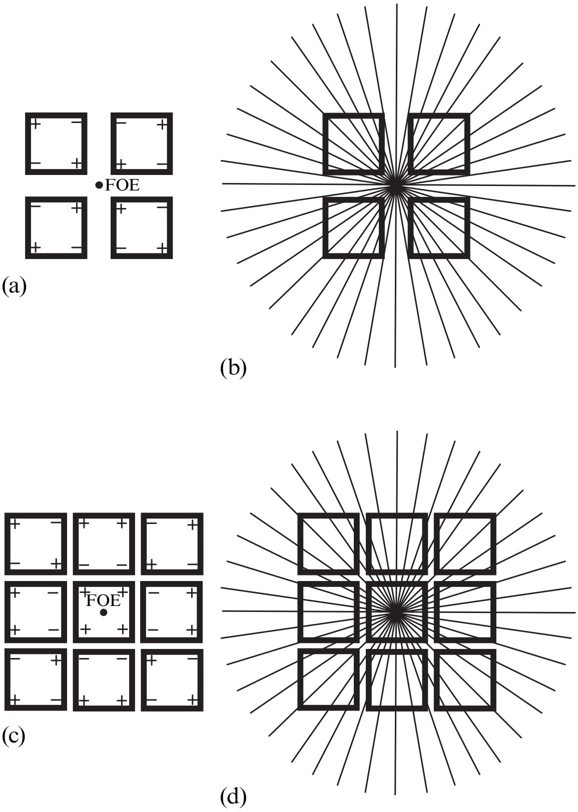


Figure 7. (a) Four squares built out of x and y lines, placed around the focus of expansion (FOE), with '+'s and '−'s summarizing the direction of projected-angle change in the next moment from figure 6. '+' ('−') indicates that the angle increases (decreases) in the next moment, and thus the predicted misperception is positive (negative). (b) The same four squares are embedded in a proximal stimulus with cues suggesting that the squares are built out of x and y lines and suggesting where the direction of motion is (the vanishing point). The reader can see that the actual directions of misperception are consistent with the predicted directions of misperception. The relative magnitudes of the misperception are also consistent with that predicted in figure 6: the misperceptions are predicted to be strongest nearest the vertical and horizontal meridians through the focus of expansion, and greatest near the focus of expansion. Finally, the absolute magnitude of the predicted misperceptions, which has a maximum around $2°$, is of the same order of magnitude as the actual misperception. (c) and (d) are analogous to (a) and (b), respectively, but for nine squares.

5 Perception of angular size and angular distance for proximal stimuli with cues they are in a plane perpendicular to the direction of motion

How do the projections of the sides of a doorway (y lines) change as an observer approaches the doorway? When an observer is far away from the doorway, the projections of the two sides are near the focus of expansion and are nearly parallel to one another (see figure 2). As an observer approaches the doorway, however, two key changes occur. First, because each projected line gets further from the focus of expansion, the angular separation between the sides increases. This can be seen in the middle viewing sphere in figure 2, where the two y line projections nearest the focus of expansion change, in time, to the two y line projections next-nearest the focus of expansion. Second, the angular separation between the regions of the projected doorway sides above and below the focus of expansion increases more slowly than the angular separation between the regions nearest the focus of expansion. The middle sphere in figure 2 also demonstrates this. Thus, although the projected doorway sides are always visually vertical, they become progressively less parallel to one another. These horizontal changes to the projected y lines are quantitatively characterized in figure 8a, which shows the change in angular distance from the vertical meridian as a function of the position of an object on a plane 1 m in front and perpendicular to the direction of motion. The main features to notice are (i) that points nearer the horizontal meridian horizontally expand more quickly away from the focus of expansion, and (ii) that the rate of expansion falls off gradually as one moves horizontally out toward the peripheral regions. Analogous observations hold for the projections of x lines. If the probable scene of a proximal stimulus consists of a number of y lines in the frontoparallel plane, and there are cues as to where the focus of expansion is, the 'perceiving the present' hypothesis expects that the perceived projections of the y lines will be not as they actually project in the proximal stimulus, but as they will probably project in the next moment. Such a scene can be constructed by placing vertical lines over a display of projected z lines converging to a vanishing point, and is shown in figure 8b. As the reader can see—and as was the case for the more than fifteen people to whom I have shown figure 8b—the projected lines are misperceived in a manner broadly consistent with the predicted directions and magnitudes from figures 8a.

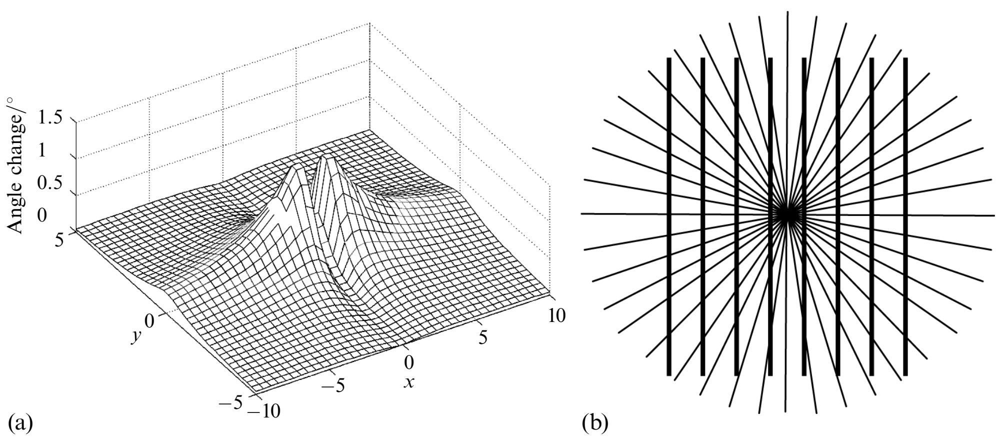


Figure 8. (a) The horizontal change in angular distance of a point from the vertical meridian, as a function of the position on a plane 1 m ahead and perpendicular to the direction of motion. The focus of expansion is at the origin. (b) Figure with cues the vertical lines are y lines, and cues as to the direction of motion. The reader can see that the y lines are perceived to project in the way they will after forward movement toward the focus of expansion, as predicted in (a).

6 Discussion

One may distinguish between two kinds of perception. One kind is *consistent perception* in which an observer perceives a scene that could have caused the proximal stimulus. For example, when an observer perceives concave indentations as convex bumps it is (usually) a case of consistent perception (albeit, in this case, a misperception); eg convex bumps with lighting from above would lead to the same proximal stimulus as concave indentations with lighting from below. The other kind is *inconsistent perception* in which an observer perceives a scene that could *not* have caused the proximal stimulus. For example, in ambiguous projected-angle perception, an observer perceives the projected angle to be nearer to 90° than it actually is; if the actual scene were this way, then it could not have generated the proximal stimulus the observer actually received. All the perceptual phenomena studied in this paper were cases of inconsistent perception.

The usual contemporary Bayesian approach to perception (Knill and Richards 1996) has achieved considerable successes in explaining perception: eg the perception of 3-D shape (Freeman 1994), binocular depth (Nakayama and Shimojo 1992; Anderson 1999), motion (Kitazaki and Shimojo 1996), lightness (Knill and Kersten 1991) and surface color (Brainard and Freeman 1997). However, the approach implicitly assumes that the visual system elicits a percept of a scene that *could have caused the proximal stimulus* (and, in particular, a scene that probably caused the proximal stimulus). For this reason, the usual contemporary Bayesian approach can accommodate only consistent perception; it cannot explain inconsistent perception. My proposal amounts to a small but significant and natural modification to the usual contemporary Bayesian approach, a modification that provides a framework rich enough to begin to explain inconsistent perception within a Bayesian framework. The modification is simply that rather than the underlying hypothesis being that, given the proximal stimulus, an observer perceives its probable source, the new, more natural hypothesis is that, given the proximal stimulus, an observer perceives the probable scene present at the time the percept is actually elicited. (Note that consistent perception is expected to occur only when the perceived attribute does not typically undergo much change in the next moment.) This new framework has allowed explanations for inconsistent perceptions, including (i) the misperception of ambiguous projected angles, (ii) the misperception of projected angles with cues the 3-D sources lie in a plane parallel to the direction of motion ($xy-z$ angles), (iii) the misperception of projected angles with cues the 3-D sources lie in a plane perpendicular to the direction of motion ($x-y$ angles), and (iv) the misperception of angular distances for lines with cues their sources lie in a plane perpendicular to the direction of motion (x lines and y lines).

Acknowledgements. I wish to thank Timothy Barber, W G Hall, Gregory Lockhead, Dan McShea, Romi Nijhawan, Surajit Nundy, Dale Purves, Dan Ryder, Christopher Tyler, Zhi-Yong Yang, and two referees for their comments and criticisms.

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